CURVILINEAR FLOW IN SOME WATER STRUCTURES

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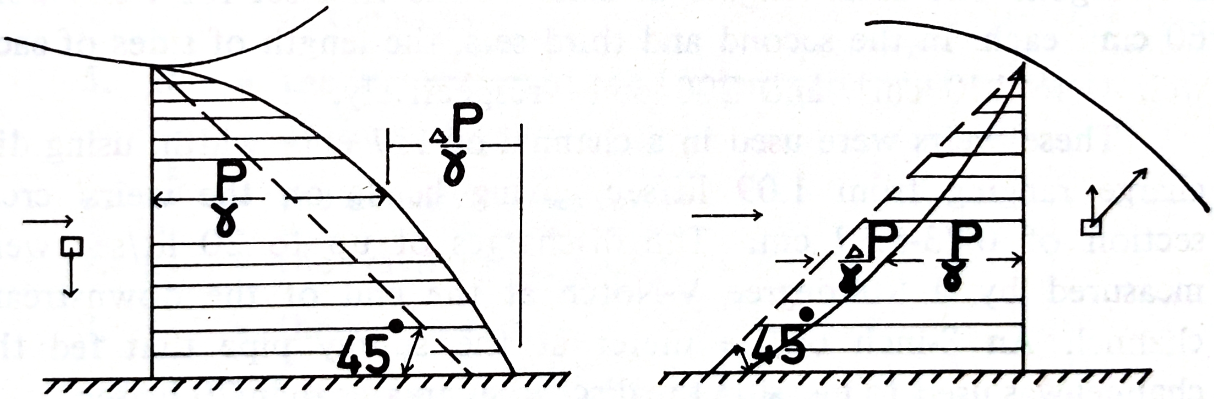
In curvilinear flow the pressure distribution along any cross section is no longer hydrostatic. The curvature produces the centrifugal forces or appreciable acceleration components normal to the direction of flow that cause a difference between the actual pressure and the hydrostatic pressure. It may be convex or concave (Fig. 1).



Pressure distribution in curvilinear flow

FIG. 1

It should be noted that the phenomena of curvilinear flow in open channel was treated successfully in the two-dimensional case, taking the curvature and inclination of tangents to the streamlines into consideration, e.g. flow over some hydraulic structures such as sharp edged weirs and broad crested weirs. The curvilinear flow over sharp edged circular escapes, which is a three-dimensional case, was also investigated.

Focus of Study

In this study, the flow over some polygonal hydraulic structures such as a sharp-edged equilateral triangle, square and hexagon weirs are investigated to find:

1. A method for calculating the coefficient of discharge of "fall-distributers" – hydraulic structures used in irrigation networks of newly-reclaimed lands, and
2. A more accurate method for designing "fall-distributers", taking into account the effect of the geometrical shape in the plan of the polygonal weir.

"Fall-distributers" reduce the water level to irrigate some low areas downstream and also to distribute water to the smaller canals which in turn feed the field. In designing them – which in plan are polygonal weirs, usually rectangular – they are usually considered to be composed of a system of straight-crested weirs.

These were represented in the laboratory by three sets of polygonal weirs, sharp crested – an equilateral triangle, a square and a hexagon. The total lengths of sides of the first set for weirs were 60 cm each. In the second and third sets, the length of sides of each weir were 120 cm and 200 cm, respectively.

These weirs were used in a channel of 117 cm width, using discharge ranging from 1.09 lit/sec, giving heads on the weirs' crest section of 0.73-5.59 cm. The discharges of up to 20 lit/sec were measured by a 90 degree V-Notch at the end of the downstream channel. An 8-inch orifice meter at the supply pipe that fed the channel was used to measure the discharges greater than 20 lit/sec.

Theoretical Consideration

The flow over a polygonal weir, which is set in a channel (Fig. 2), is not a pure radial flow; the channel sides have an effect of modifying the pattern of flow.

Therefore, the theoretical treatment of the flow over a closed polygon composed of "n" number of sides (Fig. 3), and whose internal angles are less than 180 degrees and which is set in a wide channel (side effect is neglected), is as follows: The flow over such a polygon is regarded as the flow over a system of straight stretches connected at the corners by circular arcs (Fig. 4).

Three kinds of regular polygons shall be dealt with, namely: the equilateral triangle, the square, and the hexagon. Some theoretical trends concerning other regular polygons such as pentagon, octagon, etc., may be traced in the discussion and can be expected to be comfortably generalized for all regular polygons by further investigations.

Dimensional Analysis

To start with, we have to determine the dimensionless factor that involves all the parameters affecting the discharge. In this problem, it is assumed that the amount of discharge depends on the following:

1. H = the total head
2. n = the number of sides
3. θ = the interior angles on the corners
4. ρ_s = the radius of curvature of the streamlines at the surface
5. ϕ = the inclination of the tangents to the streamlines at the surface
6. ρ = the density of the fluid
7. g = the gravitational acceleration
8. L_s = the length of side

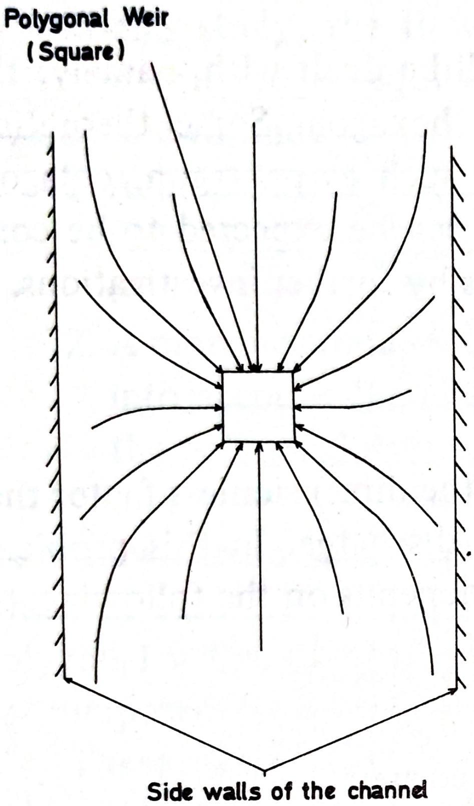
$$Q = f\,(H,\ n,\ \theta, \rho_s,\ \phi_s, \rho,\ g,\ L_s) \qquad (1)$$

By applying the II Theorem, we get:

$$\phi\left(\frac{1}{n}, \frac{1}{\theta}, \frac{H}{s}, \frac{1}{\phi}, \frac{Q^2}{gH^5}, \frac{H}{L_s}\right)$$

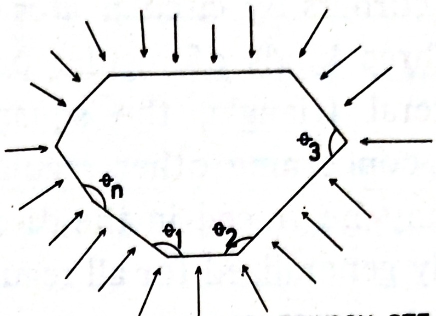
Therefore:

$$Q = \phi = \sqrt{gH}\ H^2\ \left(\frac{1}{n}, \frac{1}{\theta}, \frac{H}{\rho_s}, \frac{1}{\phi_s}, \frac{H}{L_s}\right) \qquad (3)$$



SKETCH SHOWING THE VIRTUAL FLOW OF WATER OVER A POLYGONAL WEIR IN A CHANNEL.

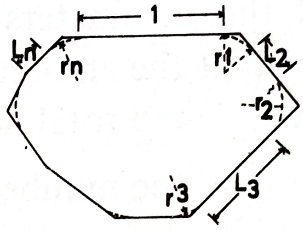
FIG. 2



SKETCH FOR FLOW OVER ANY POLYGON SET IN A WIDE CHANNEL

θ= interior corner angle.

FIG. 3



SKETCH FOR ANY POLYGONAL WEIR SHOWING THE STRAIGHT AND CURVED PARTS.

L = Length of straight part.

r = radius of circular arc

FIG. 4

From equation 3, it is clear that the amount of discharge depends upon the number of sides, the interior angles at the corners, the radius of curvature, the inclination of tangents, the total head and the ratio (H/L_s), and the Froude Number.

It is also seen that:

$$\frac{Q^2}{gH^5} = \phi\left(\frac{1}{n}\ \frac{1}{\theta}\ \frac{H}{\rho_s}\ \frac{1}{\phi_s}\ \frac{H}{L_s}\right) \qquad (4)$$

But the Froude Number:

$$Fr = \sqrt{\frac{Q^2}{gH^5}} = \frac{v}{\sqrt{gH}}$$

It will also be noted that when the geometrical shapes are given, n and θ will be known, likewise when (H/L_s) is known, then (H/ρ_s) and $(1/\phi_s)$ can be determined.

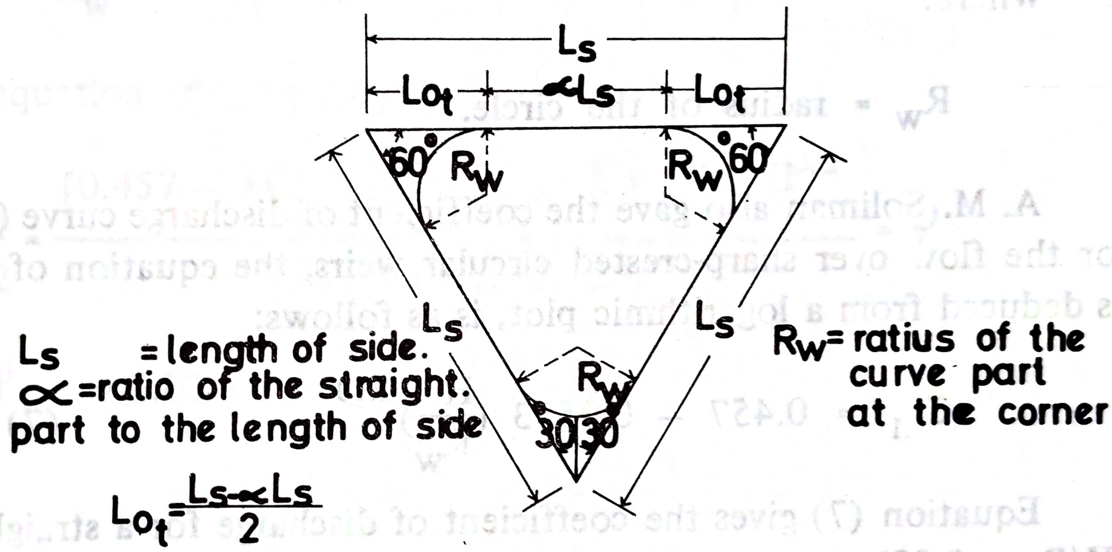
Therefore:

$$Q_p = \phi^{\equiv} \left(Fr, \frac{H}{L_s}\right) \qquad (5)$$

Where Q_p = discharge over any polygon.

Derivation of the discharge formula

A. EQUILATERAL TRIANGLE



SKETCH OF THE TRIANGLE WEIR SHOWING THE STRAIGHT AND CURVED PARTS.

FIG. 5

From Fig. 5, Let:

L_s = length of side

$\alpha L_s = L_{straight}$ = length of the straight part at a given head H.

R_w = radius of the circular arc at the corner for a given head H.

$Lo_t = \frac{L_s(1-\alpha)}{2}$ = half the difference between L_s and L_s.

Let also the discharge under a given total head, H, over one third of a circle at the corner be Q_1; the length of the corner arc be 1 (small letter L).

According to A. Khafagi and A. M. Soliman, Q_1 is equal to Q_2 is the discharge of flow over a straight weir of length 1 under the same total head H, and from the equation:

$$\psi = 1.00 - 0.116 \left(\frac{H}{R_w}\right)^{1.167} \qquad (6)$$

Where:

R_w = radius of the circle.

A. M. Soliman also gave the coefficient of discharge curve (Fig. 6) for the flow over sharp-crested circular weirs, the equation of which, as deduced from a logarithmic plot, is as follows:

$$C_{R1} = 0.457 - 0.0513 \left(\frac{H}{R_w}\right)^{1.10} \qquad (7)$$

Equation (7) gives the coefficient of discharge for a straight weir ($H/R_w = 0.00$) as to be equal to 0.457, i.e., $C_{straight} = 0.457$.

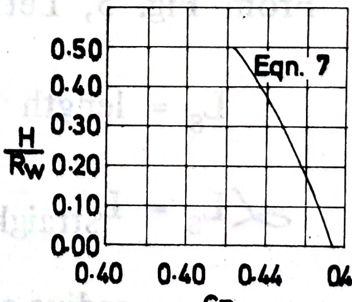
And in general, the formula for a flow over a sharp crested weir is:

$$Q = C L \times \sqrt{2g}\, H^{3/2} \qquad (8)$$

C = coefficient of discharge

L = length of the weir

H = total hand



H/R_w against C_{R1}

After M.A. Soliman

FIG. 6

The equation in the case of the equilateral triangle can be set as follows:

$$Q_1 = [0.457 - 0.0513 (\frac{H}{R_w})^{1.10}] \times L \times \sqrt{2g}\ H^{3/2} \quad (9)$$

$$Q_2 = 0.457 - L \times \sqrt{2g}\ H^{3/2} \quad (10)$$

Where:

$$R_w = Lo_t \tan 30^o = 0.577\ Lo_t$$

From equation (6), we have:

$$\frac{Q_1}{Q_2} = \frac{[0.457 - 0.0513 (\frac{H}{R_w})^{1.10}] \times L \times \sqrt{2g}\ H^{3/2}}{0.457 \times L \times \sqrt{2g}\ H^{3/2}} = \Psi$$

$$\text{But } \Psi = 1.00 - 0.116 (\frac{H}{R_w})^{1.167}$$

Therefore:

$$\frac{[0.457 - 0.513 (\frac{H}{0.577\ Lo_t})^{1.10}]}{0.457} = 1.000 - 0.116 (\frac{H}{0.577\ Lo_t})^{1.167}$$

From which we get:

$$\frac{H}{Lo_t} = 0.354 \quad (11)$$

But $Lo_t = \frac{L_s (1 - \alpha)}{2}$, and substituting in equation (11),

we get:

$$\alpha_t = 1.00 - 5.64 (\frac{H}{L_s})$$

Where: α_t = ratio of the straight part from the total length of a side of a polygon in the shape of an equilateral triangle at a given relative head (H/L_s).

Again, the flow over the triangle will be equal to the flow over three stretches of straight lines, each of lengths αL_s plus the flow over three thirds of circles, of radius 0.577 Lo_t

From Fig. (5), we have:

$$Q = 0.457 \times 3\alpha_t \times L_s \times \sqrt{2g}\ H^{3/2}$$

$$+ \left[0.457 - 0.0513 \left(\frac{H}{0.577 Lo_t}\right)^{1.10}\right]$$

$$\times 2\,\pi \times 0.577 \times Lo_t \times \sqrt{2g}\ H^{3/2}$$

But from equations (11) and (12):

$$\frac{H}{Lo_t} = 0.354 \quad \text{and} \quad \alpha_t = 1.100 - 5.64 \left(\frac{H}{L_s}\right)$$

Then substituting, we have:

$$Q_t = 0.457 \times 3 \left[1.00 - 5.64 \left(\frac{H}{L_s}\right)\right] \times L \times \sqrt{2g}\ H^{3/2}$$

$$+ \left[0.457 - 0.0513 \left(\frac{0.354}{0.577}\right)^{1.10}\right]$$

$$\times 2 \times 0.577 \times \frac{H}{0.354} \times \sqrt{2g}\ H^{3/2} \qquad (13)$$

But, Q_t = discharge over the triangle.

$$= C_t \times 3\, L_s \times \sqrt{2g}\ H^{3/2} \qquad (14)$$

Where:

C_t = coefficient of discharge over the triangle.

Equating equations (13) and (14), we have:

$$C_t \times 3L_s \times \sqrt{2g}\ H^{3/2} = 0.457 \times 3\ [1.00 - 5.64\ (H/L_s)]$$

$$\times L_s \times \sqrt{2g}\ H^{3/2} + [0.457 - 0.0513\ (\frac{0.354}{0.577})^{1.10}\ ]$$

$$\times\ 2\ \Pi \times 0.577 \times \frac{H}{0.354} \times \sqrt{2g}\ H^{3/2}$$

And $C_t = 0.457\ [1.00 - 5.64\ (\frac{H}{L_s})] \times [0.457 - 0.0513\ (0.614)^{1.10}\ ]$

$$\times \frac{2}{3} \Pi \times 1.631\ (\frac{H}{L_s})$$

Therefore:

$$C_t = 0.457 - 2.58\ (\frac{H}{L_s}) \qquad (15)$$

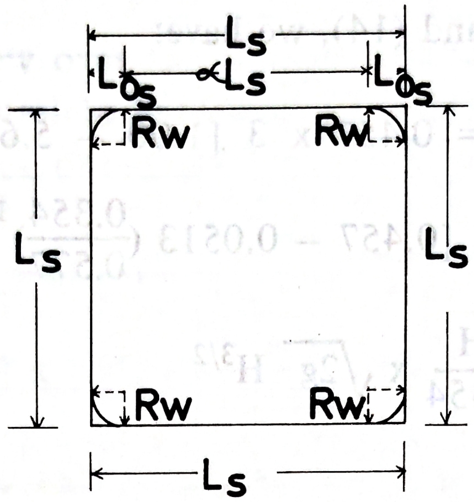
Equation (15) gives $C_t = 0.457$ when $(H/L_s) = 0.00$, i.e., when L_s is very big compared to H, then the term (H/L_s) is very small and can be neglected, in other words, for long sides, the corner effect can be neglected and the whole length of side can be treated as a straight weir.

Square

From Fig. 7, the same approach is used. In the case of the square weir, the number of sides will be four instead of three and the angle is 90^o instead of 60^o in the case of the triangle, and we have Los $L_s\ (\frac{1 - \alpha}{2})$, the index "s" refers to the square instead of index "t" which refers to the triangle in the previous section.

SKETCH OF THE SQUARE WEIR SHOWING THE STRAIGHT AND CURVED PARTS

FIG. 7



Ls = Length of side.

∝ = Ratio of the straight part to the length of side.

$Lo_s = \frac{L_s + \propto L_s}{}$

Rw = Radius at the arc at corners.

From equation (6), we have:

$$\frac{Q_1}{Q_2} = \frac{[0.457 - 0.0513\,(\frac{H}{R_w})^{1.10}] \times L \times \sqrt{2g}\;H^{3/2}}{0.457 \times L \times \sqrt{2g}\;H^{3/2}} = \Psi$$

But $\Psi = 1.000 - 0.116\,(\frac{H}{R_w})^{1.167}$

Where:

$$R_w = Lo_s \tan 45^o = Lo_s$$

Therefore:

$$\frac{[0.457 - 0.0513\,(\frac{H}{Lo_s})^{1.10}]}{0.457} = 1.00 - 0.116\,(\frac{H}{Lo_s})$$

From which we get:

$$\frac{H}{Lo_s} = 0.614$$

But $Lo_s = \dfrac{L_s\,(1 - \alpha)}{2}$, and substituting in equation (16),

we get:

$$\alpha_s = 1.00 - 3.26\,(\frac{H}{Ls})$$

Again, referring to Fig. (7), we have the discharge formula over the square by:

$$Q_s = 0.457 \times 4\alpha L_s \times \sqrt{2g}\ H^{3/2}$$
$$+ [0.457 - 0.0513\,(\frac{H}{Lo_s})^{1.10}] \times 2\,\Pi \times Lo_s \times \sqrt{2g}\ H^{3/2}$$

But from equations (16) and (17)

$$\frac{H}{Lo_s} = 0.0614 \text{ and } \alpha_s = 1.00 - 3.26\,(\frac{H}{L_s})$$

Then substituting, we have:

$$Q_s = 0.457 \times 4\,[1.00 - 3.26\,(\frac{H}{L_s})] \times L_s \times \sqrt{2g}\ H^{3/2}$$
$$+ 0.457 - 0.0513\,(0.614)^{1.10}$$
$$\times 2\,\Pi \times \frac{H}{0.614} \times \sqrt{2g}\ H^{3/2} \quad (18)$$

But Q_s = discharge over the square.

$$= C_s \times 4\,L_s \times \sqrt{2g}\ H^{3/2} \quad (19)$$

Where:

C_s = coefficient of discharge over the square.

Equating equations (18) and (19), we have:

$$C_s \times 4L_s \times \sqrt{2g}\ H^{3/2} = 0.457 \times 4\ [1.00 - 3.26(\frac{H}{L_s})] \times L_s \times \sqrt{2g}\ H^{3/2}$$

$$+ [0.457 - 0.0513\ (0.614)^{1.10}\ ] \times 2\ \Pi \times \frac{H}{0.614} \times \sqrt{2g}\ H^{3/2}$$

$$\text{and } C_s = 0.457\ [1.00 - 3.26\frac{H}{L_s}] + [0.457 - 0.0513\ (0.614)^{1.10}\ ]$$

$$\times \frac{2}{4}\ \Pi \times \frac{1}{0.614} \times (\frac{H}{L_s}).$$

Therefore:

$$C_s = 0.457 - 0.398\ (\frac{H}{L_s}) \qquad (20)$$

Equation (20) gives C_s = 0.457 when (H/L_s) = 0.00, i.e., when L_s is very big compared to H, then the term (H/L_s) is very small and can be neglected, and the whole length of side can be treated as a straight weir.

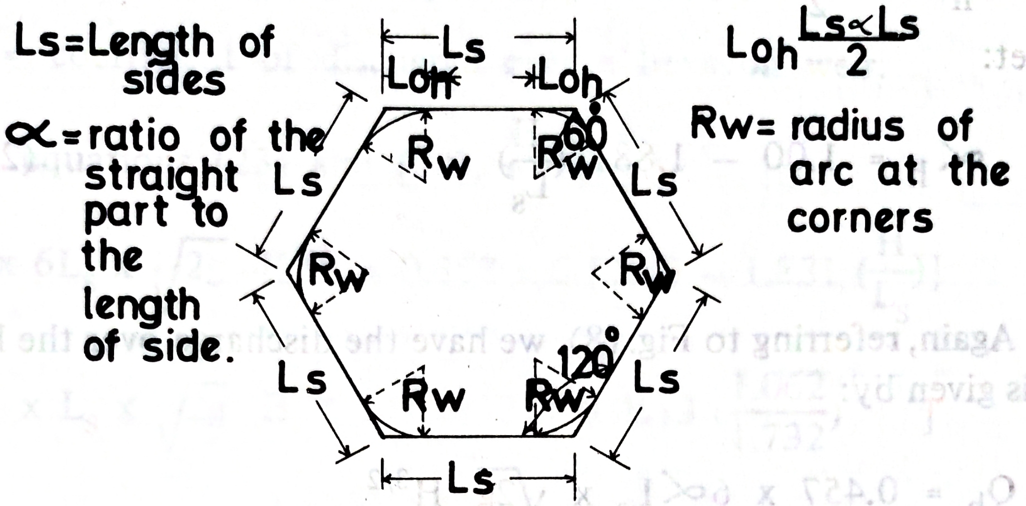
Hexagon

From Fig. (8):

Number of side, n = 6

Corner angle, θ – 120 degrees.

$Lo_h = \frac{L_s(1 - \alpha)}{2}$, the index "h" refers to the hexagon.



SKETCH OF THE HEXAGONAL WEIR SHOWING THE STRAIGHT AND CURVED PARTS.

FIG. 8

and we have:

$R_w = Lo_h \tan 60^o = 1.732\ Lo_h$, then by the same procedure as before, we have:

$$\frac{Q_1}{Q_2} = \frac{[0.457 - 0.0513\ (\frac{H}{R_w})^{1.10}] \times L \times \sqrt{2g}\ H^{3/2}}{0.457 \times L \times \sqrt{2g}\ H^{3/2}} = \Psi$$

But $\Psi = 1.00 - 0.116\ (\frac{H}{R_w})^{1.167}$

Therefore:

$$\frac{[0.457 - 0.0513\ (\frac{H}{1.732 Lo_h})^{1.10}]}{0.457} = 1.000 - 0.116\ (\frac{H}{1.732 Lo_h})^{1.167}$$

From which we get:

$$\frac{H}{Lo_h} = 1.063$$

But, $Lo_h = \frac{L_s(1 - \alpha)}{2}$, and substituting in equation (21), we get:

$$\alpha_h = 1.00 - 1.881 \left(\frac{H}{L_s}\right) \qquad (22)$$

Again, referring to Fig. (8), we have the discharge over the hexagon and is given by:

$$Q_h = 0.457 \times 6\alpha L_s \times \sqrt{2g}\ H^{3/2}$$

$$+ \left[0.457 - 0.0513 \left(\frac{H}{Lo_h \tan 30^o}\right)^{1.10}\right]$$

$$\times\ 2\ \Pi \times 1.732\ Lo_h \times \sqrt{2g}\ H^{3/2}$$

But from equations (21) and (22):

$$\frac{H}{Lo_h} = 1.063 \text{ and } \alpha_h = 1.00 - 1.881 \left(\frac{H}{L_s}\right)$$

Then substituting, we have:

$$C_h = 0.457 \times 6 \left[1.00 - 1.881 \left(\frac{H}{L_s}\right)\right] \times L_s \times \sqrt{2g}\ H^{3/2}$$

$$+ \left[0.457 - 0.0513 \left(\frac{1}{1.732}\right)^{1.10}\right]$$

$$\times\ 2\ \Pi \times 1.732 \times H \times \sqrt{2g}\ H^{3/2} \qquad (23)$$

But Q_h = discharge over the hexagon.

$$= C_h \times L_s \times \sqrt{2g}\ H^{3/2} \qquad (24)$$

Where:

C_h = coefficient of discharge over a hexagon weir.

Equating equations (23) and (24), we have:

$$C_h \times 6L_s \times \sqrt{2g}\ H^{3/2} = 0.457 \times 6\ [1.00 - 1.881\ (\frac{H}{L_s})]$$

$$\times L_s \times \sqrt{2g}\ H^{3/2} + [0.457 - 0.0513\ (\frac{1.062}{1.732})^{1.10}\ ]$$

$$\times 2\ \Pi \times \frac{1.732}{1.063} \times \frac{H}{L_s} \times \sqrt{2g}\ H^{3/2}$$

and $C_h = 0.457\ [1.00 - 1.881\ (\frac{H}{L_s})] + [0.457 - 0.513\ (\frac{1.063}{1.732})]^{1.10}$

$$\times \frac{2}{6}\ \Pi \times \frac{1.732}{1.063} \times (\frac{H}{L_s})$$

Therefore:

$$C_h = 0.457 - 0.132\ (\frac{H}{L_s}) \qquad (25)$$

Equation (25) gives C_h = 0.457 when (H/L_s) = 0.00, i.e., when the value of L_s is very big as compared to H, then the term (H/L_s) is very small and can be neglected; in other words, for long sides, the corner effect can be neglected, and the whole length of the side can can be treated as a straight weir.

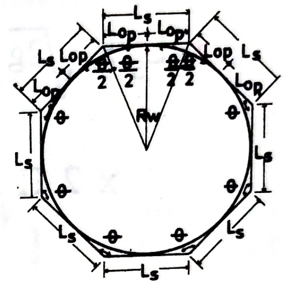
For other regular polygons

It is noted in the previous sections that for a given total head H, the ratio (H/Lo) is increasing with the number of sides "n". It is also noted that as we increase the number of sides the corresponding ratio (H/R_w) for the polygon approaches that for the circular weir.

This gives us a sign to a certain trend that we can generalize the expression for the ratio (H/Lo), which can be called the "Polygonal Factor" (P.F.).

General derivations

From Fig. (9), let:



Any regular polygon
(e. g. octagon)
FIG. 9

n = number of sides of a polygon.

θ = interior corner angle.

α_p = ratio of the straight part to the total length of side of the polygon.

$$Lo_p = \frac{L_s(1-\alpha)}{2}$$

L_s = length of side of the polygon.

R_w = radius of the inscribed circle in the polygon.

$= Lo_p \tan(\Theta/2)$

Q_1 = discharge through a circular weir with a circumference equal to L.

Q_2 = discharge through a straight weir with a total length equal to L.

$$\psi = \frac{Q_1}{Q_2} = 1.00 - 0.116\left(\frac{H}{L_w}\right)^{1.167}$$

$$C_{RI} = 0.457 - 0.0513\left(\frac{H}{R_w}\right)^{1.10}$$

C_p = coefficient of discharge of any polygon.

For any regular closed polygon;

$$Q_1 = C_{RI} \times L \times \sqrt{2g}\ H^{3/2}$$

$$= [0.457 - 0.0513 (\frac{H \quad 66}{\tan\frac{\theta}{2}\ Lo_p})^{1.10}] \times L \times \sqrt{2g}\ H^{3/2}$$

$$Q_2 = 0.457 \times L \times \sqrt{2g}\ H^{3/2}$$

Therefore:

$$\frac{Q_1}{Q_2} = \frac{[0.457 - 0.0513 (\frac{H}{\tan(\theta/2) Lo_p})^{1.10}] = \times L \times \sqrt{2g}\ H^{3/2}}{0.457 \times L \times \sqrt{2g}\ H^{3/2}} = \Psi$$

But, $\Psi = 1.00 - 0.116 (\frac{H}{Lo_p \tan(\theta/2)})^{1.167}$

Therefore we get:

$$\frac{H}{Lo_p} = 0.614 \tan(\theta/2)$$

Or, $$\text{P. F.} = 0.614 \tan [\frac{(n-2)}{2n} \times 180^o] \quad (26)$$

Also a general expression for the coefficient of discharge through any Regular Polygon, C_p can be given as follows:

$$C_p = 0.457 - (\frac{0.914}{\text{P. F.}}) - (\frac{4.373}{n}) (\frac{H}{L_s})$$

It is in the terms of the Polygonal Factor, P.F. and the number of sides n, whose corresponding values are found in Fig. (10) or Table I, or by equation (26).

Again, the general equation for the discharge over a regular polygon can be written as follows:

$$Q - C_p \times n\ L_s \times \sqrt{2g}\ H^{3/2}$$

n = no. of sides

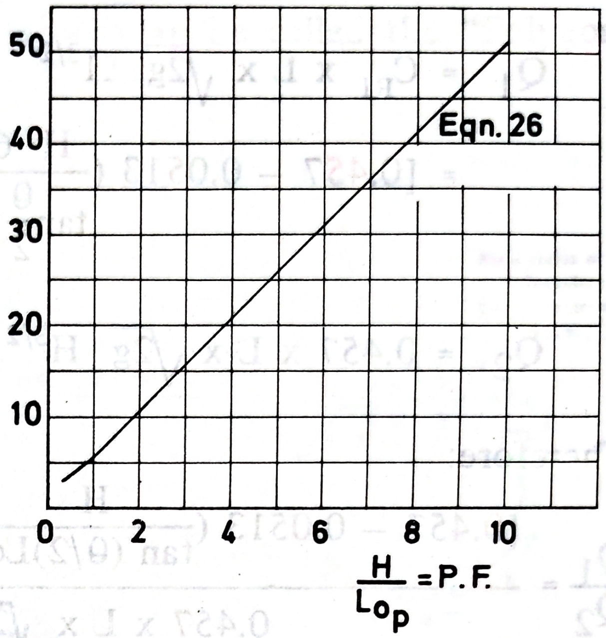
H = total head

$L_{op} = \frac{L_s (1-\alpha)}{2}$

P. F. = polygon factor

No. of side n against polygon factor P. F.

FIG. 10



Where:

C_p = coefficient of discharge over a polygonal weir.

n = number of sides of the regular polygonal weir.

L_s = length of the side of the polygonal weir.

H = total head.

and $C_p = 0.457 - x\left(\frac{H}{L_s}\right)$ (29)

Now, Q equals zero when C_p equals zero, i.e., the nappes over the sides interfers with each other and the weir flow breaks. This leads us to the idea of having the ratio of (H/L_s) at which the weir flow breaks. Putting $C_p = 0.000$ in equation (29), we have:

$$\left(\frac{H}{L_s}\right)_z = \frac{0.457}{x} \qquad (30)$$

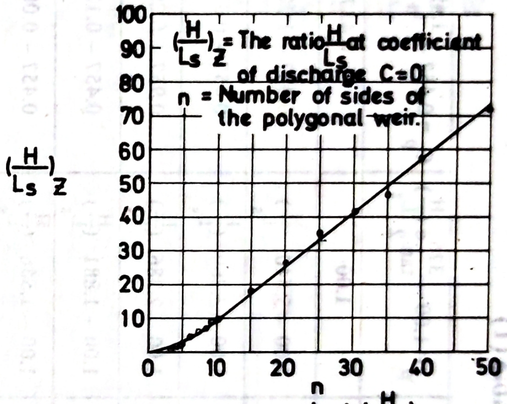
Where:

$(\frac{H}{L_s})_Z$ = the ratio of (H/L_s) at which the weir flow breaks, the index Z refers to this stage.

For some other regular polygon, the corresponding $(\frac{H}{L_s})_Z$ was calculated (see Table I, also see curve, Fig. 11).

Again, if we consider the case for the regular polygon of 50 sides (n = 50, see Table I), as to represent practically the case of the circle, we have:

$$2 \pi R_w = n L_s = 50 L_s$$



n against $(\frac{H}{L_s})_Z$

FIG. 11

Where:

R_w = the radius of the considered circle.

n = the number of sides of the polygon which is considered practically representing the circle.

L_s = the length of the side of this polygon.

Then we have:

$$L_s = \frac{2 \pi R_w}{50} \qquad (31)$$

But the equation for the flow over the polygon is:

$$C_p = 0.457 - 0.0064 \left(\frac{H}{L_s}\right), \text{ (see Table I)}$$

Table (1)

n = no. of sides	interior θ = corner angle	$\tan\frac{\theta}{2}$	$H_{Lo_p} 0.674 \tan\frac{\theta}{2}$ = P.F.	$p = 1.00 - \frac{326}{\tan\frac{\theta}{2}} (\frac{H}{L_s})$	$C_p = 0.457 - \frac{1.491}{\tan\frac{\theta}{2}} - \frac{4.373}{n} \frac{H}{L_s}$	$(\frac{H}{L_s})$ for $C_p = 0$
1	0°	0	0	1.00	0.457	0
3*	6°	0.577	0.354	$1.00 - 5.640 (\frac{H}{L_s})$	$0.457 - 1.128 (\frac{H}{L_s})$	0.405
4*	90°	1.000	0.614	$1.00 - 3.260 (\frac{H}{L_s})$	$0.457 - 0.398 (\frac{H}{L_s})$	1.148
5	108°	1.376	0.845	$1.00 - 2.386 (\frac{H}{L_s})$	$0.957 - 0.209 (\frac{H}{L_s})$	2.185
6*	120°	1.732	1.063	$1.00 - 1.881 (\frac{H}{L_s})$	$0.457 - 0.132 (\frac{H}{L_s})$	3.460
7	128.57°	2.096	1.284	$1.00 - 1.556 (\frac{H}{L_s})$	$0.457 - 0.087 (\frac{H}{L_s})$	5.250
8	135°	2.414	1.482	$1.00 - 1.350 (\frac{H}{L_s})$	$0.457 - 0.071 (\frac{H}{L_s})$	6.440
9	140°	2.748	1.687	$1.00 - 1.184 (\frac{H}{L_s})$	$0.457 - 0.054 (\frac{H}{L_s})$	8.460
10	144°	3.078	1.890	$1.00 - 1.058 (\frac{H}{L_s})$	$0.457 - 0.048 (\frac{H}{L_s})$	9.520
15	156°	4.705	2.890	$1.00 - 0.692 (\frac{H}{L_s})$	$0.457 - 0.025 (\frac{H}{L_s})$	18.30
20	162°	6.314	3.875	$1.00 - 0.516 (\frac{H}{L_s})$	$0.457 - 0.017 (\frac{H}{L_s})$	26.85
25	165.6°	7.930	4.870	$1.00 - 0.411 (\frac{H}{L_s})$	$0.457 - 0.013 (\frac{H}{L_s})$	35.15
30	168°	9.514	5.840	$1.00 - 0.3925 (\frac{H}{L_s})$	$0.457 - 0.0111 (\frac{H}{L_s})$	41.15
35	169.72°	11.060	6.790	$1.00 - 0.2945 (\frac{H}{L_s})$	$0.457 - 0.0099 (\frac{H}{L_s})$	46.15
40	171°	12.706	7.810	$1.00 - 0.256 (\frac{H}{L_s})$	$0.457 - 0.0080 (\frac{H}{L_s})$	57.10
50**	172.8°	15.880	9.750	$1.00 - 0.205 (\frac{H}{L_s})$	$0.457 - 0.0064 (\frac{H}{L_s})$	71.40
					$(\frac{H}{R_w}) = 8.974$	

* Investigated Polygons

** See Explanation on page 19

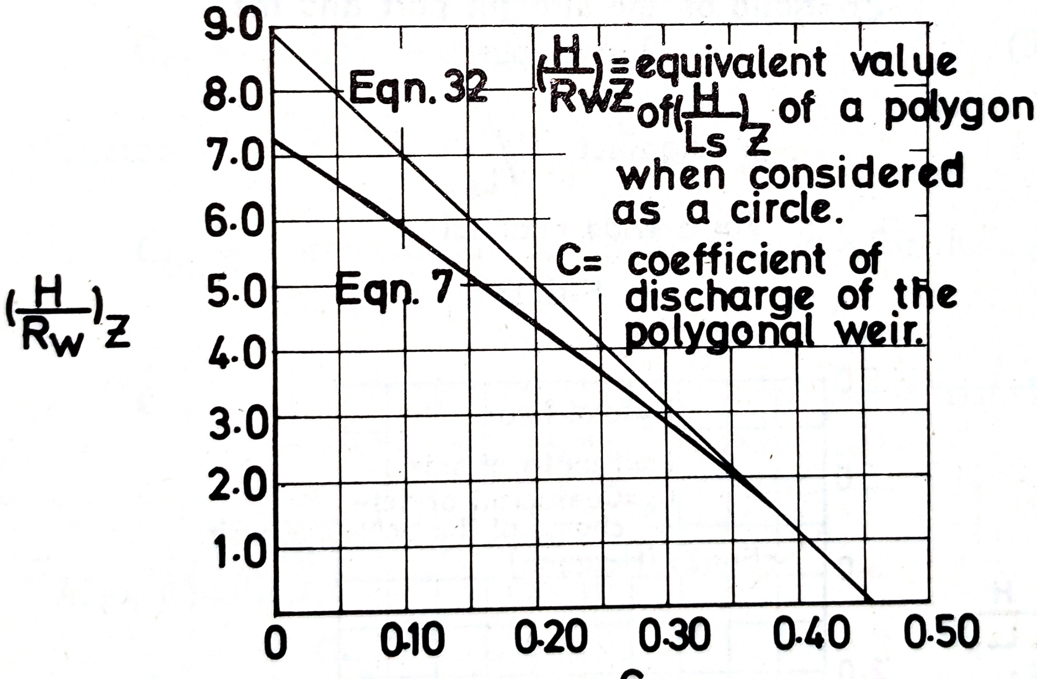
From equation (30), we have:

$$C_p = 0.457 - 0.0064 \left(\frac{H \times 25}{\pi R_w}\right)$$

$$C_p = 0.0457 - 0.051 \left(\frac{H}{R_w}\right) \quad (32)$$

$$\text{But, } C_R = 0.457 - 0.513 \left(\frac{H}{R_w}\right)^{1.10} \quad (7)$$

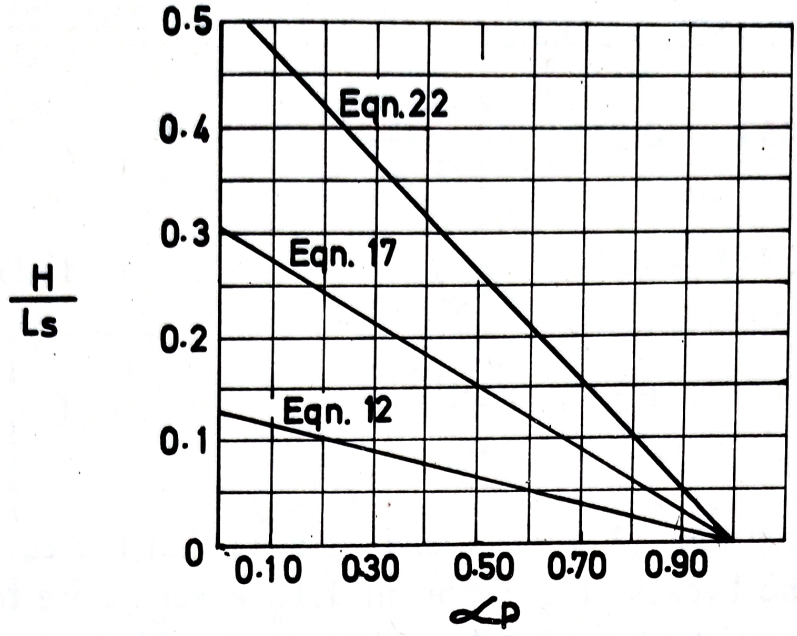
Comparing equations (32) and (7), it is clear that the equations are almost the same because the exponent 1.10 is very close to 1.00 and their curves will virtually coincide with the range of the weir flow (Fig. 12).



C against $(\frac{H}{R_w})_z$

FIG. 12

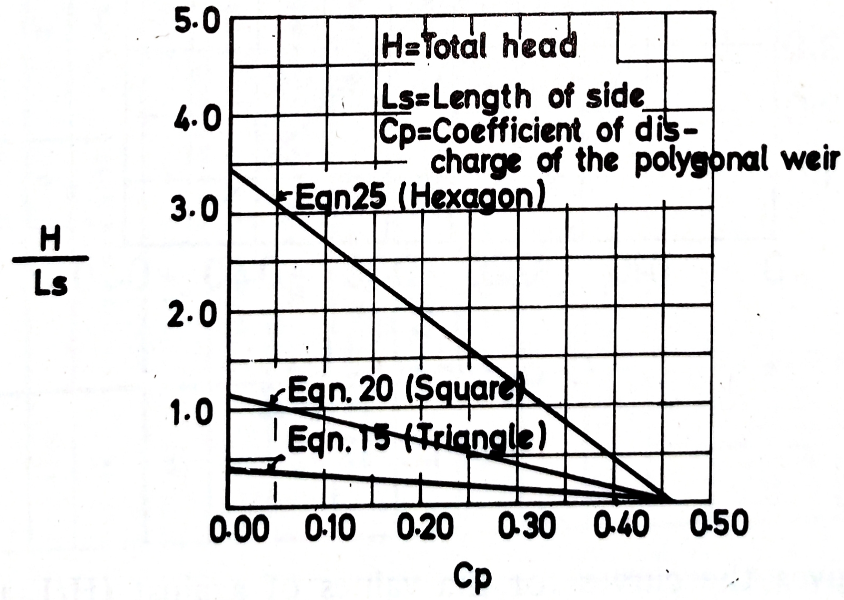
Fig. 13 gives the curves for the values of against (H/L_s), while Fig. 14 gives the curves for the values of C_p against (H/L_s).



H= Total head.
Ls =Length of a side.
∝p =Ratio of the straight part and the curved part.

∝p Against H/Ls
For a wide channel

FIG. 13



Cp Against H/Ls

FIG. 14

Analysis of Data

It is clear in Fig. 15 that in the set of small models whose L_T = 60 cms. and 120 cms., the difference between the theoretical lines and the lines passing through the experimental points is so small, except in the triangle where the sharp corner angle has a pronounced effect, especially at high heads.

Inasmuch as the theoretical values and the experimental results differ a correction factor C_F is deduced from the experiments to correct the values of the coefficients of discharge (Fig. 16).

For the equilateral triangle weir:

Equation (15) when corrected can be written as follows:

$$C_{tc} = [0.457 - 1.128 (\frac{H}{L_s})] C_{Ft} \quad (33)$$

Where:

C_{tc} = corrected coefficient of discharge for the equilateral triangle weir.

C_{Ft} = correction factor for the equilateral triangle model as deduced from the experiments.

Also, equation (12) can be written as:

$$\alpha_{tc} = \frac{C_{tc}}{0.200C_{Ft}} = 1.284 \quad (34)$$

Where:

α_{tc} = the ratio of the straight part to the length of side taking into consideration the correction factor C_{Ft} for the equilateral triangle weir.

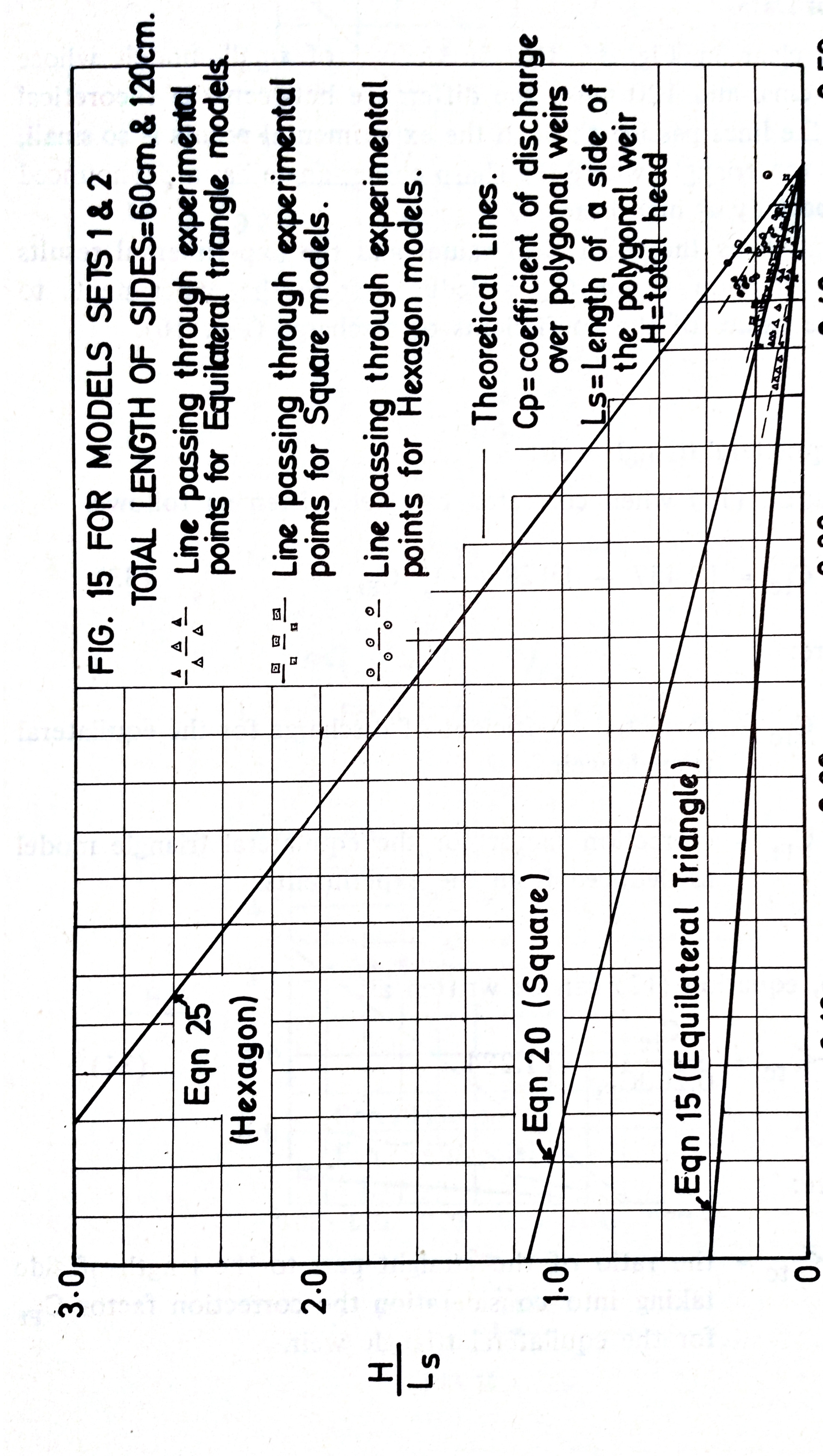
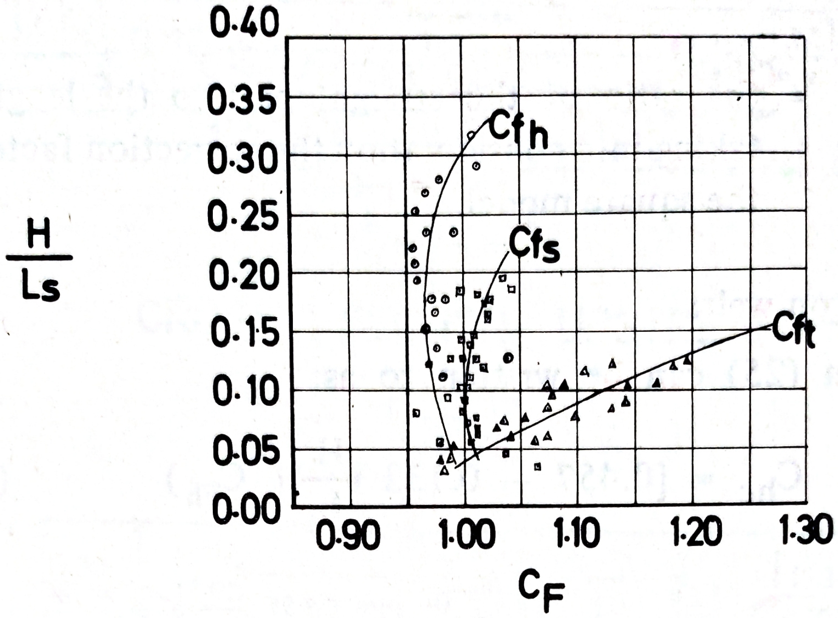


FIG. 15 FOR MODELS SETS 1 & 2
TOTAL LENGTH OF SIDES=60cm.& 120cm.
Line passing through experimental points for Equilateral triangle models.
Line passing through experimental points for Square models.
Line passing through experimental points for Hexagon models.
Theoretical Lines.
Cp=coefficient of discharge over polygonal weirs
Ls=Length of a side of the polygonal weir
H=total head
Eqn 25 (Hexagon)
Eqn 20 (Square)
Eqn 15 (Equilateral Triangle)
H/Ls
3.0
2.0
1.0
0
0.10
0.20
0.30
0.40
0.50



Cf_t = Correction factor for Equilateral triangle weir.

Cf_s = Correction factor for Square weir.

Cf_h = Correction factor for Hexagon weir.

FIG. 16

For the square weir:

Equation (20) can be written also as:

$$C_{sc} = [0.457 - 0.398 (\frac{H}{L_s})] C_{Fs} \qquad (35)$$

Where:

C_{sc} = corrected coefficient of discharge for the square.

C_{Ft} = correction factor for the square model as deduced from the experiments.

Also equation (17) can be written as:

$$\alpha_{sc} = \frac{C_{sc}}{0.122\ C_{Ft}} - 2.745$$

Where:

α_{sc} = the ratio of the straight part to the length of side taking into consideration the correction factor C_{Fs} for the square model.

For the hexagon weir:

Equation (25) can be written to as:

$$C_{hc} = [0.457 - 0.132 (\frac{H}{L_s})] C_{Fh}) \qquad (37)$$

Where:

C_{hc} = corrected coefficient of discharge for the hexagon weir.

C_{Fh} = correction factor for the hexagon model as deduced from experiments.

Also equation (22) can be written as:

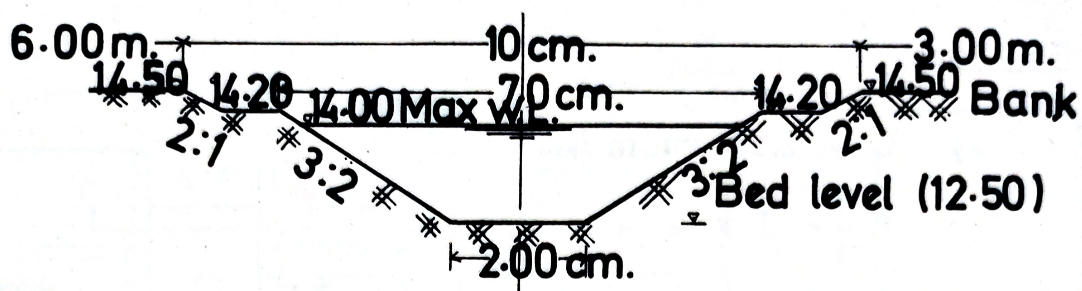
$$\alpha_{hc} = \frac{C_{hc}}{0.0702\ C_{Fh}} - 5.51 \qquad (38)$$

Where:

α_{hc} = the ratio of the straight part to the length of side taking into consideration the correction factor C_{Fh} for the hexagon model.

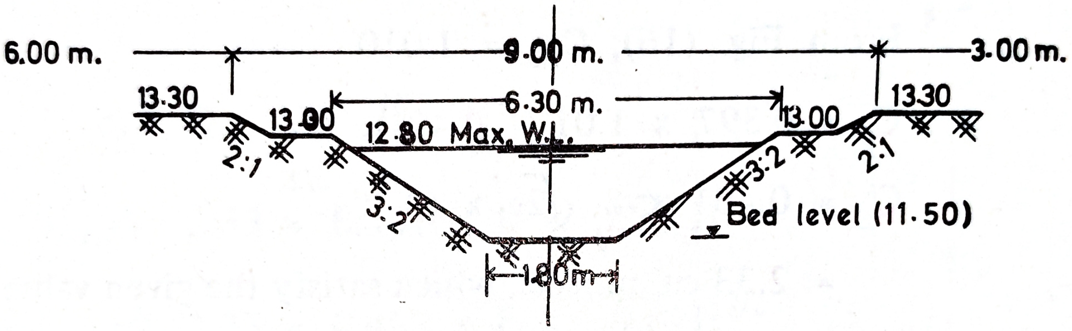
Illustrative Example

An irrigation canal's water level dropped 1.20 m at a certain alignment according to the area's topography. The cross-sections, upstream and downstream the fall, are shown in Fig. 17 and 18. To feed the two branches A and B are to be fed by two pipeline culverts



CROSS SECTION UPSTREAM

FIG. 17



CROSS SECTION DOWNSTREAM

FIG. 18

under the banks. If the discharge of the canal upstream the fall is 2.33 cu.m./sec., corresponding to the water level (Fig. 19).

a) Find the crest level of the fall distributer.

b) Find the radius of the fillet at the corners.

c) Find the error in calculating the head, if the given fall distributer is regarded as four straight weirs, each having of length 2.00 m.

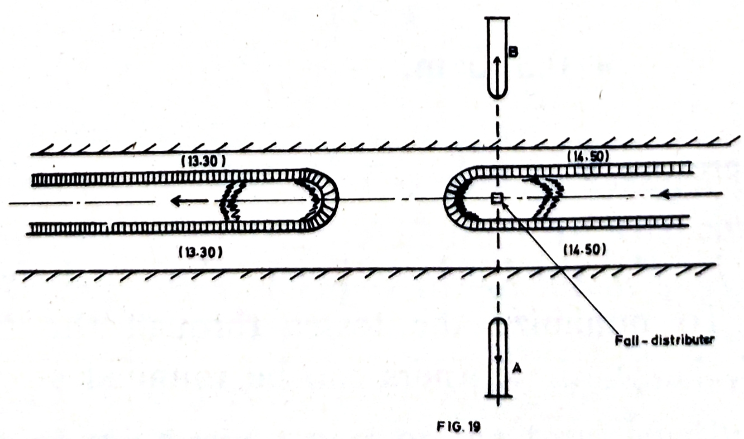


FIG. 19

Solutions:

a) $Q = 2.33$ cu. m./sec.

$L_T = 2 \times 4 - 8$ m.

Let $H = 30$ cm.

$(H/L_S) = (0.30/2.00) - 0.15$

$C_S = 0.457 - 0.398 \times 0.15 = 0.397$

From Fig. (16), $C_{Fs} = 1.010$

$C_S = 0.397 \times 1.01 = 0.401$

$Q_S = 0.401 \times 8 \sqrt{2g} \times (0.30)^{3/2}$

$= 2.33$ cu. m./sec., which satisfy the given value.

Therefore H = 30 cms.

And the crest level = 14.00 – 0.30

= 13.70 m.

b) From Equation (36):

$$\alpha_{sc} = \frac{0\ 397}{0.122} - 2.745 = 5.10$$

$$\text{But, } Lo_S = \frac{2\ (1 - \alpha)}{2} = \frac{2\ (1 - 0.510)}{2}$$

$= 0.390$ m.

Therefore $R_W = Lo_S = 39$ cm.

Fig. 20 is a longitudinal section in the canal showing the "fall-distributer." To minimize the losses through the structure at the maximum discharge, the corners can be rounded at a radius 39 cms. as in Fig. 21.

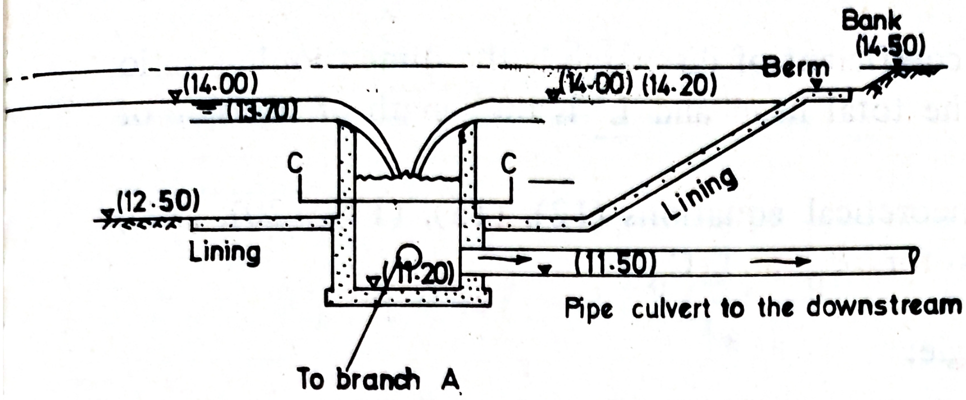


FIG. 20

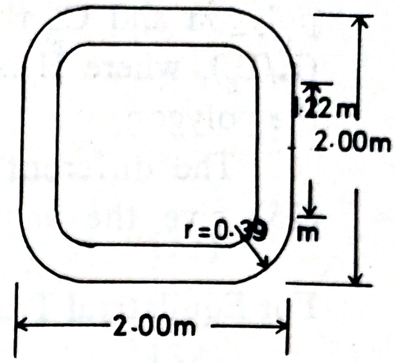


FIG. 21

c) $Q = CL\sqrt{2g}\ H^{3/2}$.

$$2.33 = 0.401 \times 8 \times \sqrt{2g}\ H^{3/2}$$

$$2.33 = 0.457 \times 8 \times \sqrt{2g}\ H^{3/2}$$

$$\left(\frac{H_1}{H_2}\right)^{3/2} = \frac{0.401}{0.457}$$

$\frac{H_1}{H_2} = 0.917$, from which the percentage of error in head equals 9.17. The percentage of error in calculating the discharge of assuming straight weirs for the whole length will be:

$$\%\ \text{error in discharge} = 1.00 - \frac{0.401}{0.457} \times 100$$

$$= 12.2\%$$

Discussion

The condition of flows over these three polygonal weirs changes due to the sharp corner and the magnitude of the corner angle θ. So, the difference of the discharges for different polygonal shapes exists.

The parameter that affected the discharges, and thus took effect on α, the ratio of the straight part of the total length of side of a polygon and C_p the coefficient of discharge is the dimensionless ratio (H/L_s), where H is the total head and L_s is the length of the side of the polygon.

The different theoretical equations (12), (15), (17), (20), (22), (25) give the values for α_p and C_p:

For Equilateral Triangle:

$$\alpha_t = 1.00 - 5.64 \left(\frac{H}{L_s}\right) \quad (12)$$

$$C_t = 0.457 - 1.128 \left(\frac{H}{L_s}\right) \quad (15)$$

For the Square:

$$\alpha_s = 1.00 - 3.26 \left(\frac{H}{L_s}\right) \quad (17)$$

$$C_s = 0.457 - 0.398 \left(\frac{H}{L_s}\right) \quad (20)$$

For the Hexagon:

$$\alpha_h = 1.00 - 1.881 \left(\frac{H}{L_s}\right) \quad (22)$$

$$C_h = 0.457 - 0.132 \left(\frac{H}{L_s}\right) \quad (25)$$

Then the final equations are:

For Triangle:

$$C_{tc} = 0.457 - 1.128 \left(\frac{H}{L_s}\right) C_{Ft} \quad (33)$$

$$\alpha_{tc} = \frac{C_t}{0.200\ C_{Ft}} - 1.284 \quad (34)$$

For Square:

$$C_{sc} = 0.457 - 0.398 \left(\frac{H}{L_s}\right) C_{Fs} \quad (35)$$

$$\alpha_{sc} = \frac{C_s}{0.122C_{Fs}} - 2.745 \quad (36)$$

For Hexagon:

$$C_{hc} = 0.457 - 1.32 \left(\frac{H}{L_s}\right) \quad (37)$$

$$\alpha_{hc} = \frac{C_h}{0.0702C_{Fh}} - 5.51 \quad (38)$$

Conclusion

As a result of this research, the various parameters affecting the coefficient of discharge for flows over an equilateral triangle, a square, and a hexagon weirs were investigated successfully and can now be calculated.

Also, the differences in the values of discharges passing over these weirs — although of the same total length L_T and affected by the same total head H — were found considerable.

Equations (12), (15), (17), (20), (22), and (25), can therefore be used for the calculations of flow in proposals to improve hydraulic structures such as "fall-distributers" in irrigation channel, especially when the channel is relatively wide with respect to the structures. Moreover, a correction factor C_F was introduced to compensate for the effect of the channel side and the corner angles (The C_F values are found in the curves in Fig. 14.)

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